

# Privacy-Preserving Collaborative Data Mining

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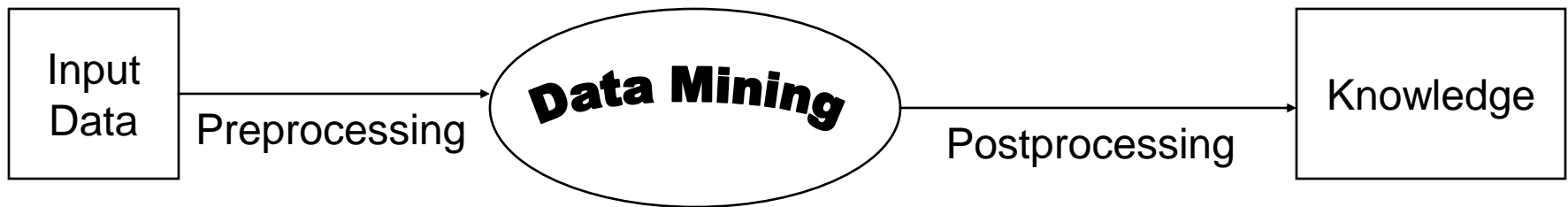
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# Overview

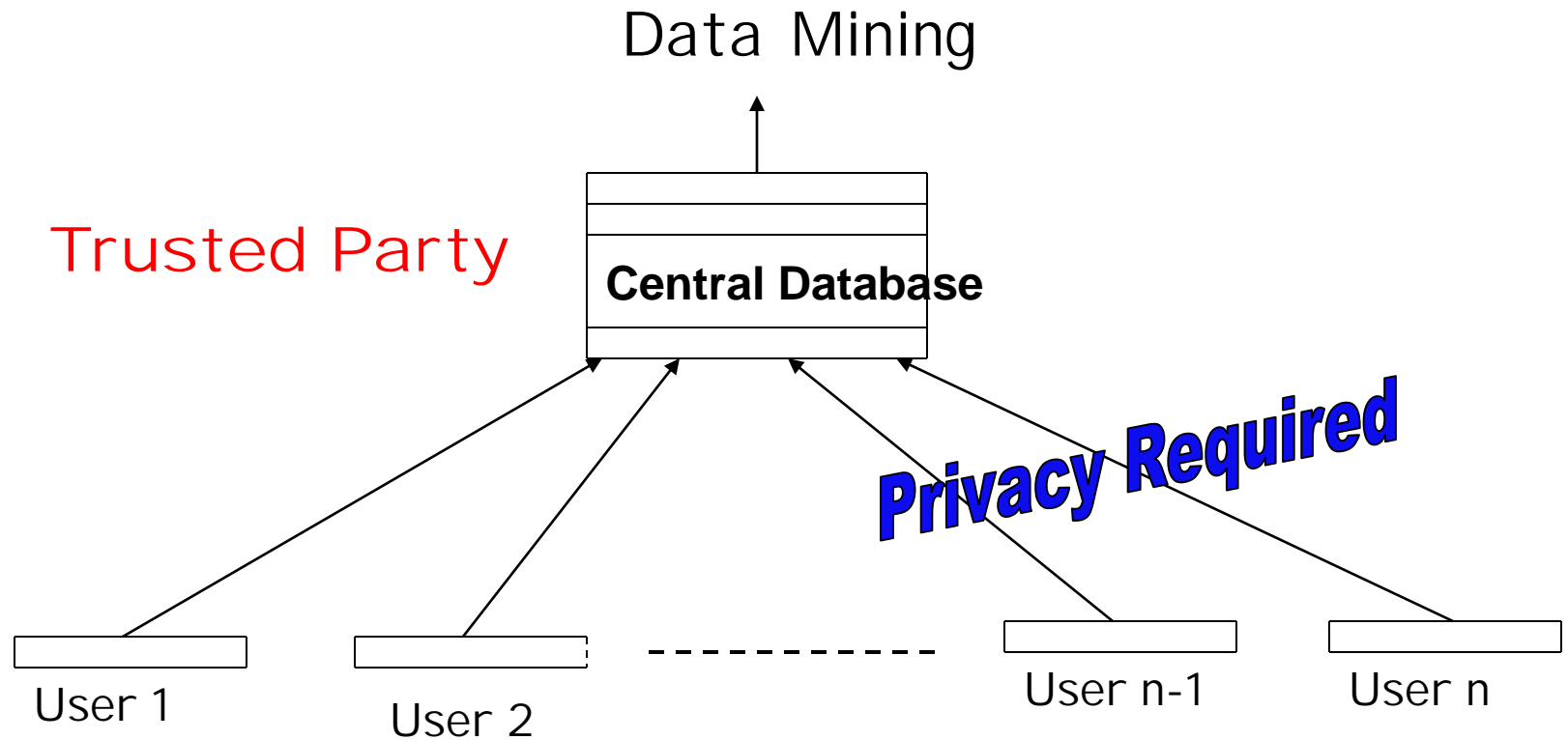
- Privacy Learning Library
- Efficient Privacy-Preserving Collaborative Compiler System Using Scalar Product
- Social Computing

# What Data Mining Is

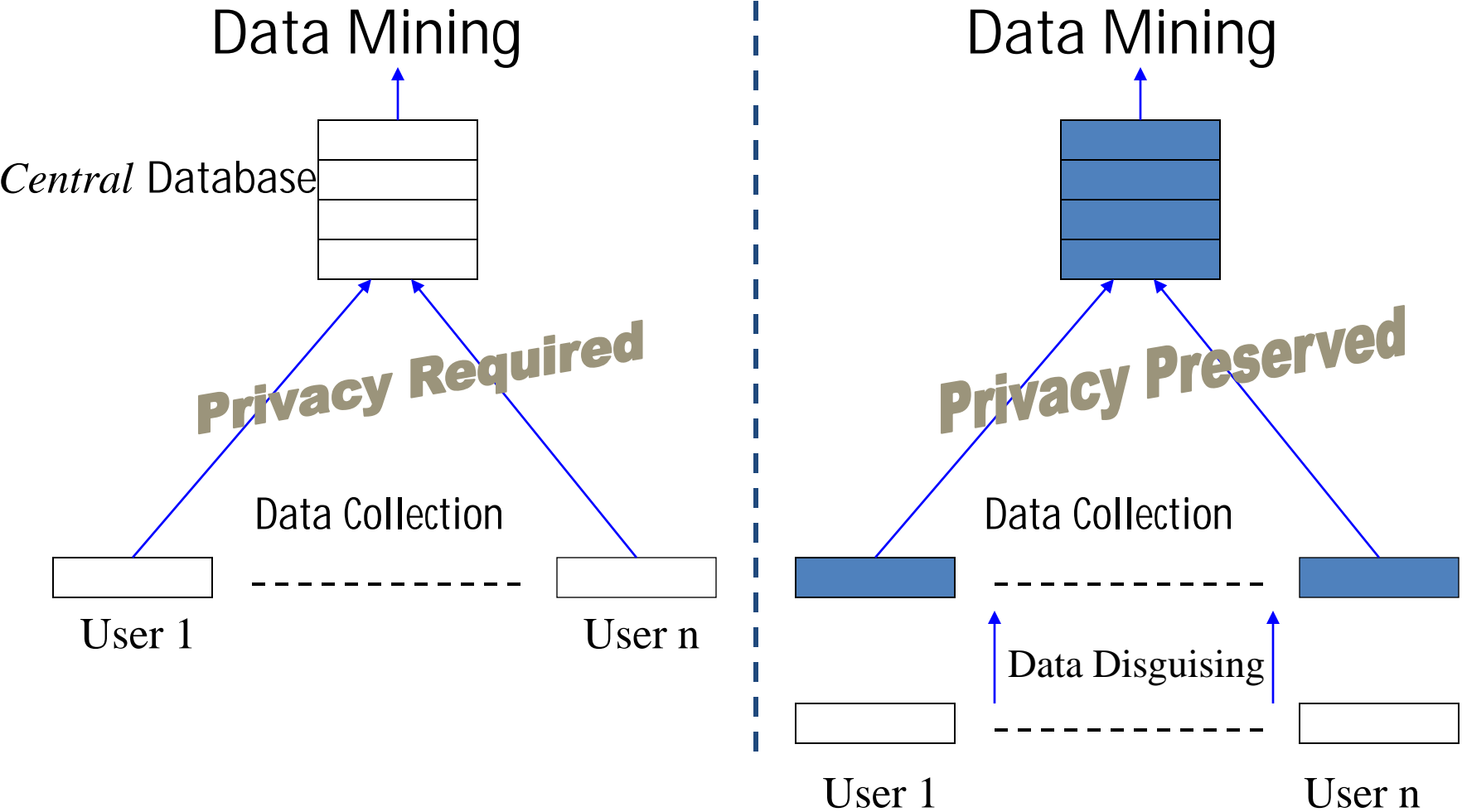
- Data mining is the process of automatically discovering useful knowledge in large databases.



# A Trusted-Party Model



# Privacy Protection



# Randomized Response Techniques

- An example:  
Survey: how many people have ever driven while intoxicated?
- People may not want to divulge their information
- How to conduct such a survey?
- Two related questions are asked for each person
  1. Is it true that you have ever driven while intoxicated?
  2. Is it true that you haven't ever driven while intoxicated?
- Each person randomly selects one question to answer
  - Probability of selecting question 1 is  $\frac{1}{2}$ .
  - Probability of selecting question 2 is  $(1 - \frac{1}{2})$ .

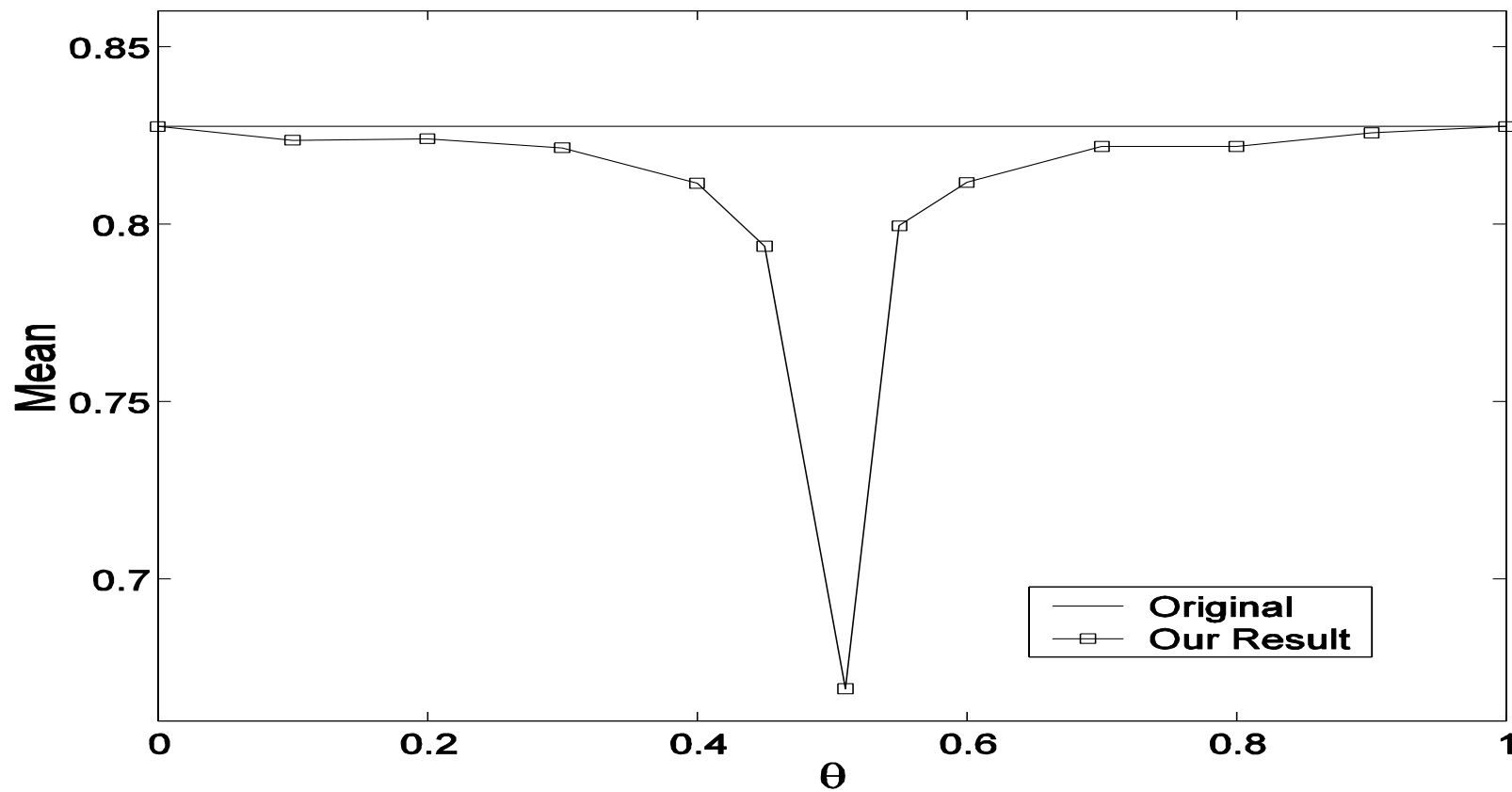
# How Randomized Response Works

$$P^*(A = \text{yes}) = P(A = \text{yes}) + P(A = \text{no})(1 - \lambda) \quad (1)$$

$$P^*(A = \text{no}) = P(A = \text{no}) + P(A = \text{yes})\lambda \quad (2)$$

- $P^*(A = \text{yes})$  and  $P^*(A = \text{no})$ : directly count the disguised data.
- $P(A = \text{yes})$ : The percentage of people who have driven while intoxicated.
- Solving Eq. (1) and (2), we get  $P(A = \text{yes})$ .

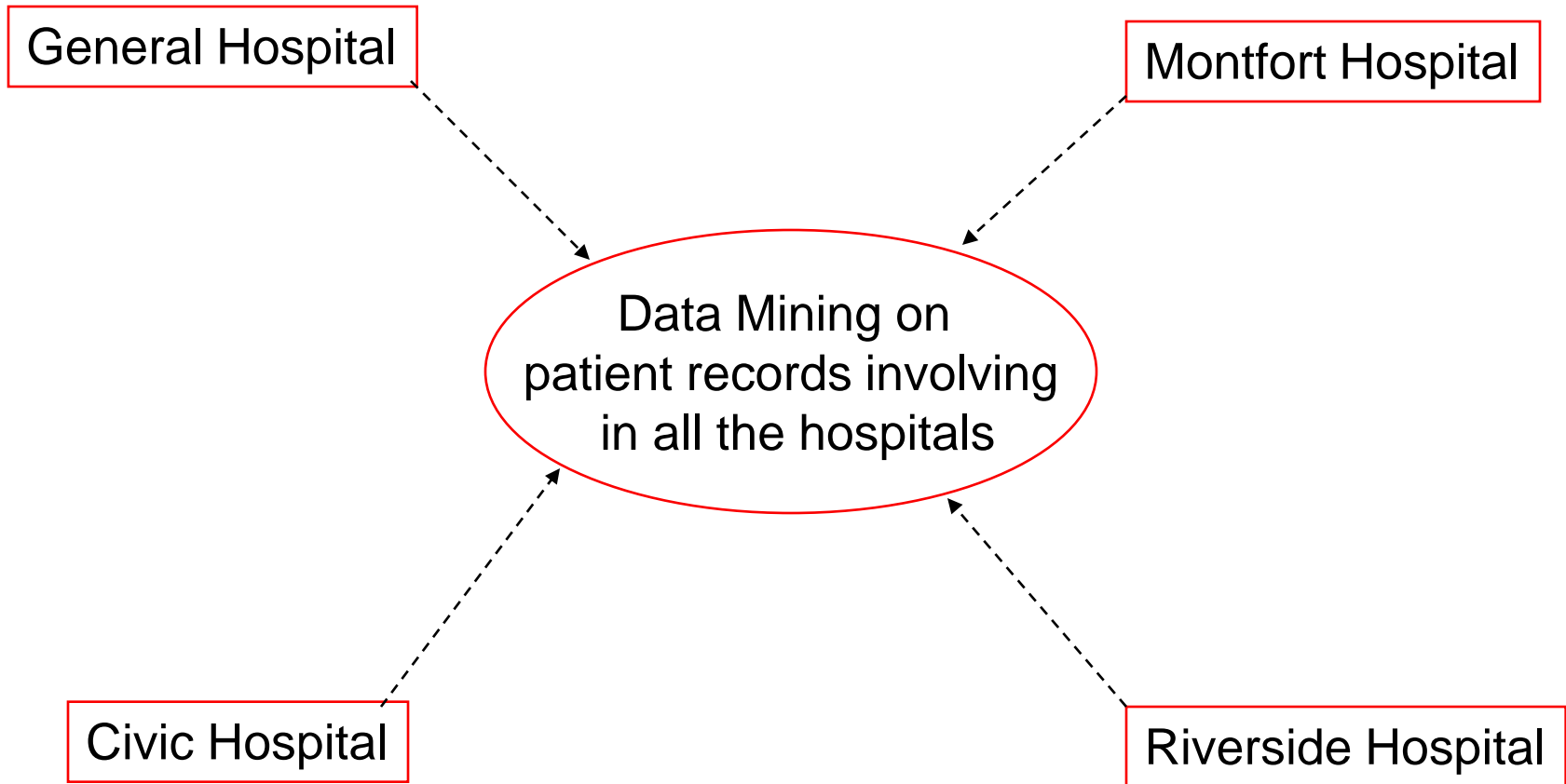
# Experimental Results



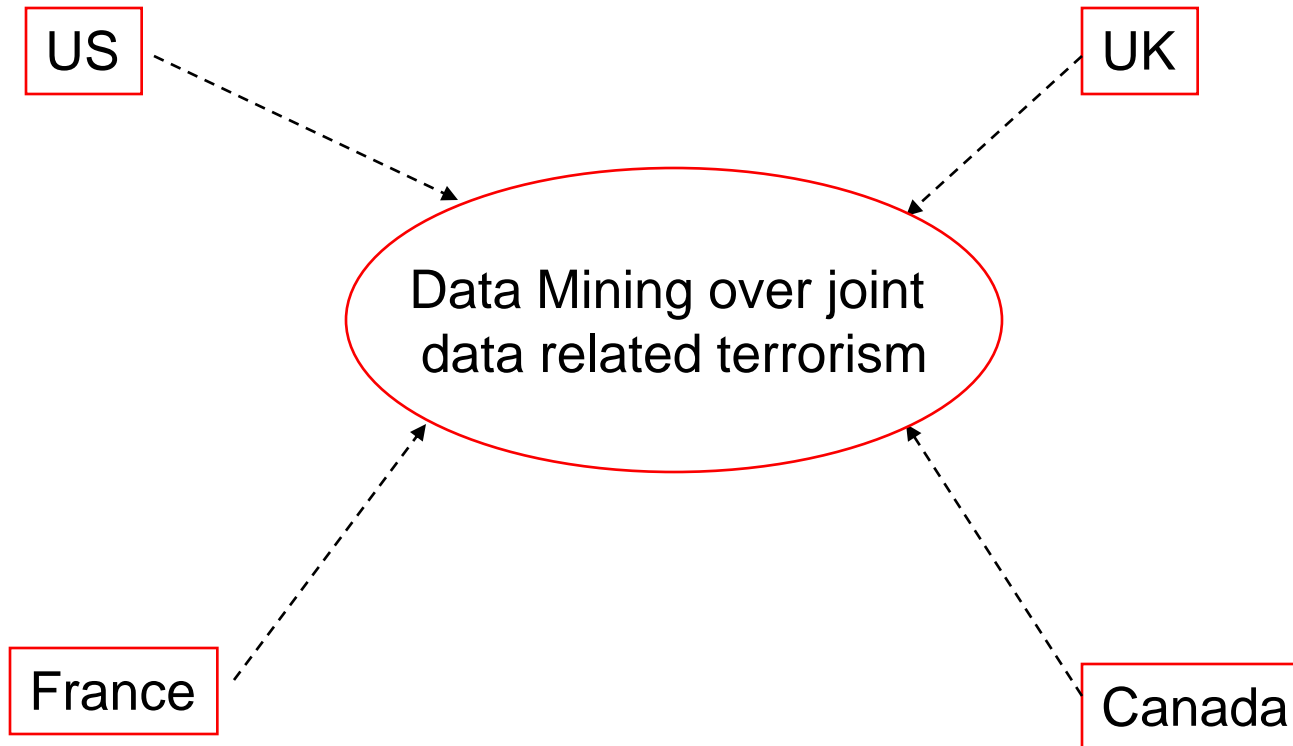


# Why PPDM is Important

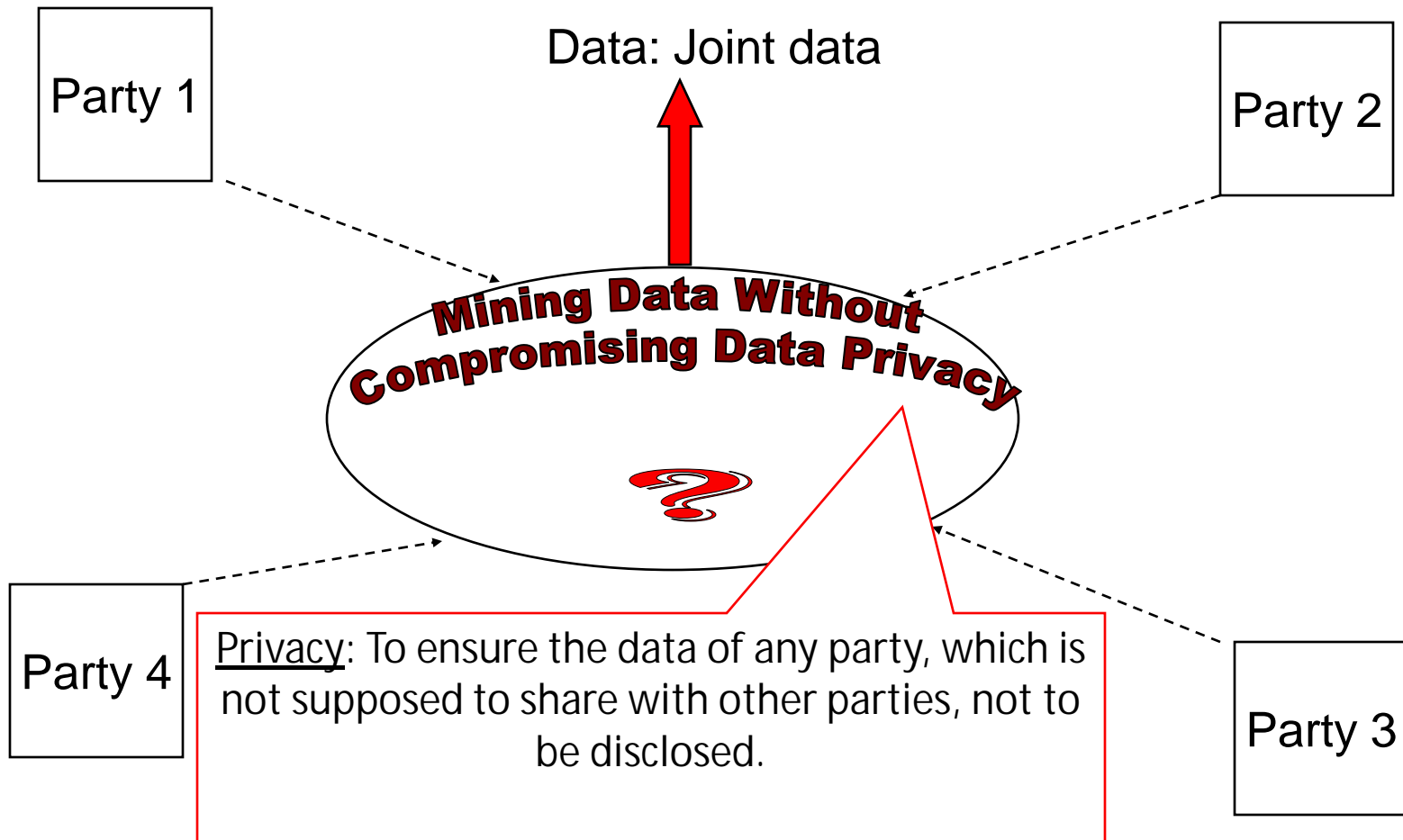
# Biomedical Computing



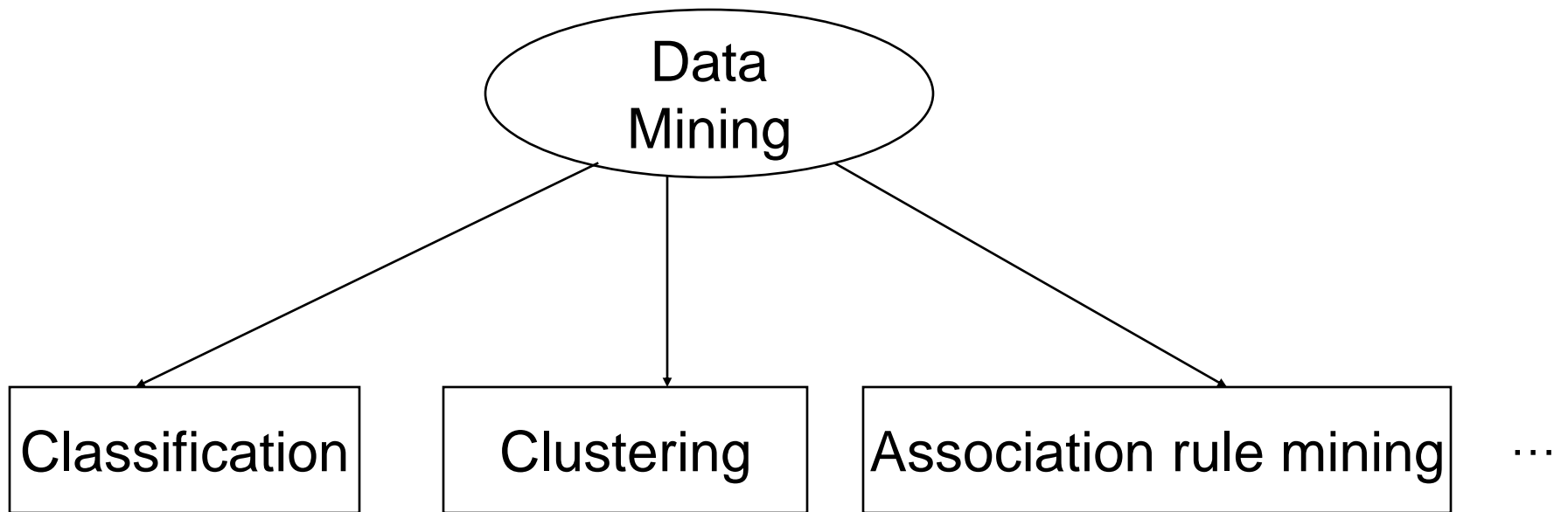
# Government Collaboration (NATO)



# Privacy-Preserving Collaborative Data Mining



# Data Mining Tasks



# Classification

Training Set

Tid	Attrib1	Attrib2	Class
1	Yes	Large	No
2	No	Medium	No
3	No	Large	Yes
4	No	Small	Yes

Learning

Learning  
Algorithm

Learn  
Model

Model

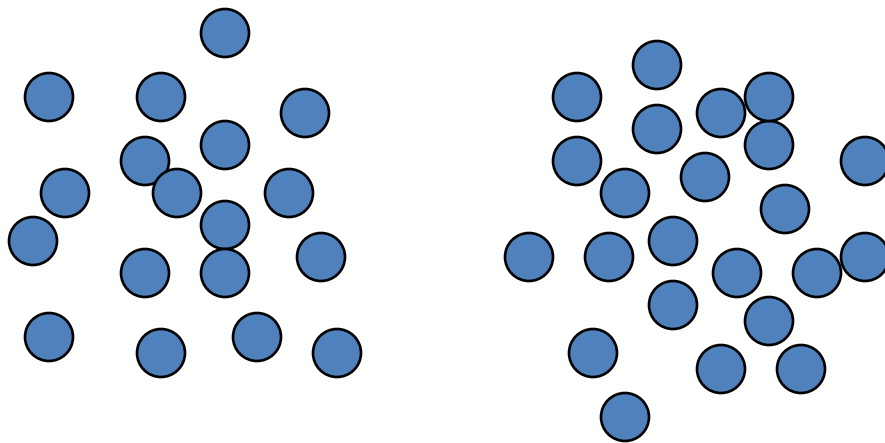
Test Set

Tid	Attrib1	Attrib2	Class
1	No	Large	?
2	Yes	Large	?

Apply  
Model

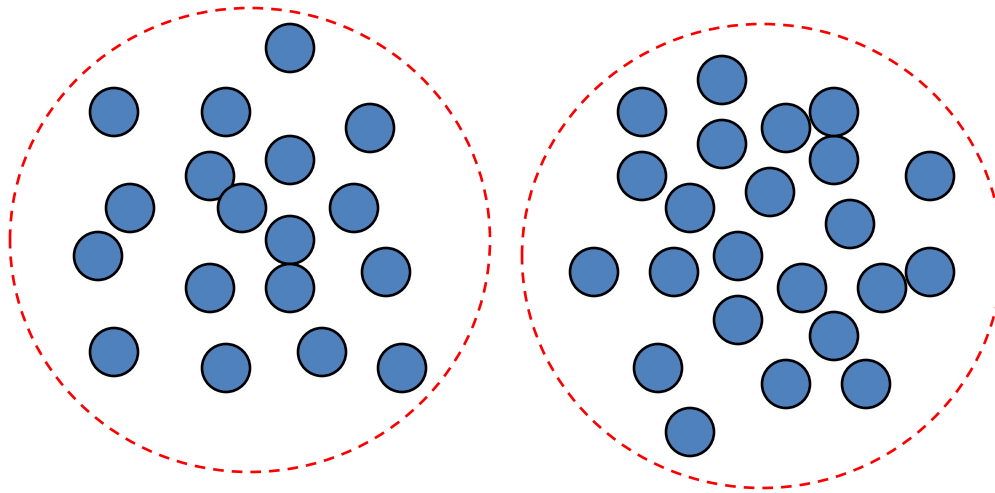
Prediction

# Clustering



**The data objects**

# Clustering

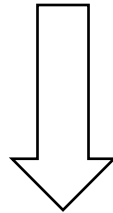


**Two clusters**

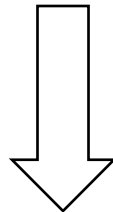


# Association Rule Mining

Wal-Mart customer transaction data



Association Rule Mining



We may discover the rule  
{Diapers} {Beer}

# Association Rule Mining

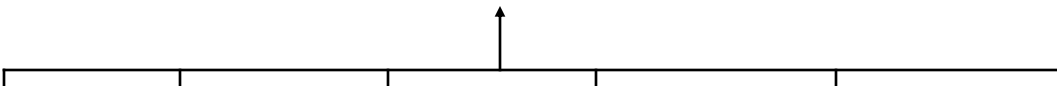
TID	Items
1	{Bread, Milk}
2	{Bread, Diapers, Beer, Eggs}
3	{Milk, Diapers, Beer, Coke}
4	{Bread, Milk, Diapers, Beer}
5	{Bread, Milk, Diapers, Coke}

An example of market basket transactions.

TID: Transaction ID

# Binary Representation

The List of Attributes



TID	Bread	Milk	Diapers	Beer	Eggs	Coke
1	1	1	0	0	0	0
2	1	0	1	1	1	0
3	0	1	1	1	0	1
4	1	1	1	1	0	0
5	1	1	1	0	0	1

# Itemset

- Let  $I = \{i_1, i_2, \dots, i_d\}$  be the set of all items in a store and  $T = \{t_1, t_2, \dots, t_N\}$  be the set of all transactions.
- Each transaction  $t_i$  contains a subset of items chosen from  $I$
- In association rule mining, a collection of zero or more items is termed an itemset.
- If an itemset contains  $k$  items, it is called a  $k$ -itemset.

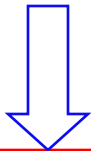
**{Beer, Diapers, Milk}**  $\longleftrightarrow$  **3-itemset**

# Association Rule

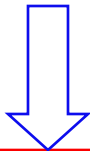
- An association rule is an expression of the form  $X \Rightarrow Y$ , where  $X$  and  $Y$  are different sets of items.

**{Bread}      {Beer}**

The strength of an association rule



Support



Confidence

# Support and Confidence

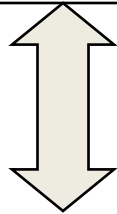
- Support determines how often a rule can be applied to a given data set.
- Confidence determines how frequently items in Y appear in transactions that contain X.

$$\textit{Support}, \quad s(X \rightarrow Y) = \Pr(X \rightarrow Y)$$

$$\textit{Confidence}, \quad c(X \rightarrow Y) = \frac{\Pr(X \rightarrow Y)}{\Pr(X)}$$

# Association Rule Mining

Given a set of transactions  $T$ , find all the rules having support  $minsup$ , and confidence  $minconf$ , where  $minsup$  and  $minconf$  are the corresponding support and confidence thresholds.



## Privacy-Preserving Collaborative Association Rule Mining

To enable multiple parties to conduct association rule mining over their joint data sets without disclosing their private data.

# Horizontal Collaboration

	TID	Bread	Milk	Diapers	Beer	Eggs	Coke
Alice	1	1	1	0	0	0	0
	2	1	0	1	1	1	0
	3	0	1	1	1	0	1
<hr/>							
Bob	4	1	1	1	1	0	0
	5	1	1	1	0	0	1



# Vertical Collaboration

**Store 1**

TID	Bread	Milk	Diapers
1	1	1	0
2	1	0	1
3	0	1	1
4	1	1	1
5	1	1	1

**Alice**

**Store 2**

TID	Beer	Wine
1	1	0
2	1	1
3	0	1
4	1	1
5	1	1

**Bob**

**This Talk Focuses on Vertical Collaboration**

# Association Rule Mining Algorithm

[Agrawal et al. 1993]

1.  $L_1$  = large 1-itemsets
2. for  $C_k$  *a priori*  $gen(L_{k-1})$  do begin
3.      $(k = 2; L_{k-1} \neq \emptyset; k++)$
4.     for all candidates  $c \in C_k$  do begin
5.         compute c.count
6.     end
7.      $L_k = \{c \in C_k \mid c.count \geq \min\_sup\}$
8. end
9. Return  $\bigcup_k L_k$

c.count is the frequency count for a given itemset.

Key issue: to compute the frequency count, we need to access attributes that belong to different parties.

# Frequent Itemset Generation

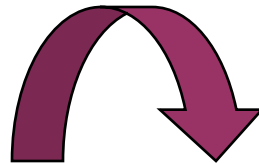
Minimum support count = 3

**Candidate  
1-Itemsets**

Item	Count
Beer	3
Bread	4
Diapers	4
Milk	4

**Candidate  
Large 1-Itemsets**

Item	Count
Beer	3
Bread	4
Diapers	4
Milk	4



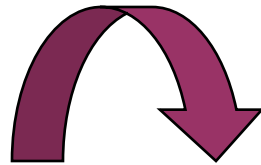
**REMOVED**

# Frequent Itemset Generation

Minimum support count = 3

**Candidate  
Large 1-Itemsets**

Item	Count
Beer	3
Bread	4
Diapers	4
Milk	4



**Candidate  
2-itemsets**

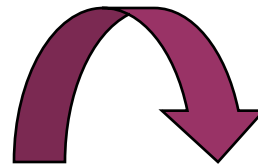
Itemset	Count
{Beer, Bread}	2
{Beer, Diapers}	3
{Beer, Milk}	2
{Bread, Diapers}	3
{Bread, Milk}	3
{Diapers, Milk}	3

# Frequent Itemset Generation

Minimum support count = 3

**Candidate  
2-itemsets**

Itemset	Count
{Beer, Diapers}	3
{Bread, Diapers}	3
{Bread, Milk}	3
{Diapers, Milk}	3



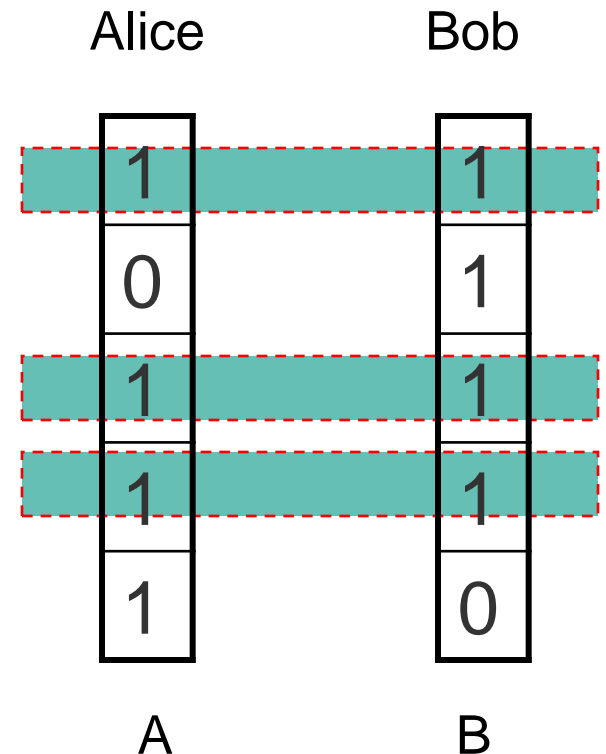
**Candidate  
Large 2-itemsets**

Itemset	Count
{Beer, Diapers}	3
{Bread, Diapers}	3
{Bread, Milk}	3
{Diapers, Milk}	3

**REMOVED**

# An Example

- $c.count$  is the vector product.
- Let's use  $A$  to denote Alice's attribute vector and  $B$  to denote Bob's attribute vector.
- $AB$  is a candidate frequent itemset, then  $c.count = A \cdot B = 3$ .
- How to conduct this computation across parties without compromising each party's data privacy?



# Homomorphic Encryption

[Paillier 1999]

- Privacy-preserving protocols are based on Homomorphic Encryption.
- Specifically, we use the following additive homomorphism property:

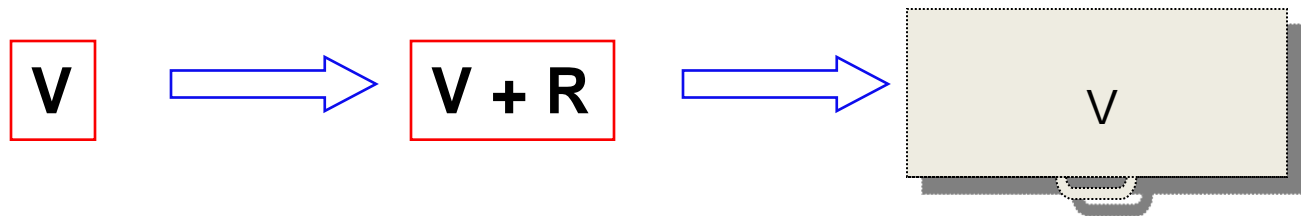
$$e(m_1) \cdot e(m_2) \cdot \dots \cdot e(m_n) = e(m_1 + m_2 + \dots + m_n)$$

- Where  $e$  is an encryption function and  $m_i$  is the data to be encrypted and  $e(m_i) \neq 0$ .

# Digital Envelope

[Chaum85]

- A digital envelope is a random number (a set of random numbers) only known by the owner of private data.





# Frequency Count Protocol

- Assume Alice's attribute vector is  $A$  and Bob's attribute vector is  $B$ .
- Each vector contains  $N$  elements.
- $A_i$  : the  $i$ th element of  $A$ .
- $B_i$  : the  $i$ th element of  $B$ .
- One of parties is randomly chosen as a key generator, e.g, Alice, who generates  $(e, d)$  and an integer  $X > N$ .  $e$  and  $X$  will be shared with Bob.
- Let's use  $e(.)$  to denote encryption and  $d(.)$  to denote decryption.

# Step 1

Alice

**Computes**

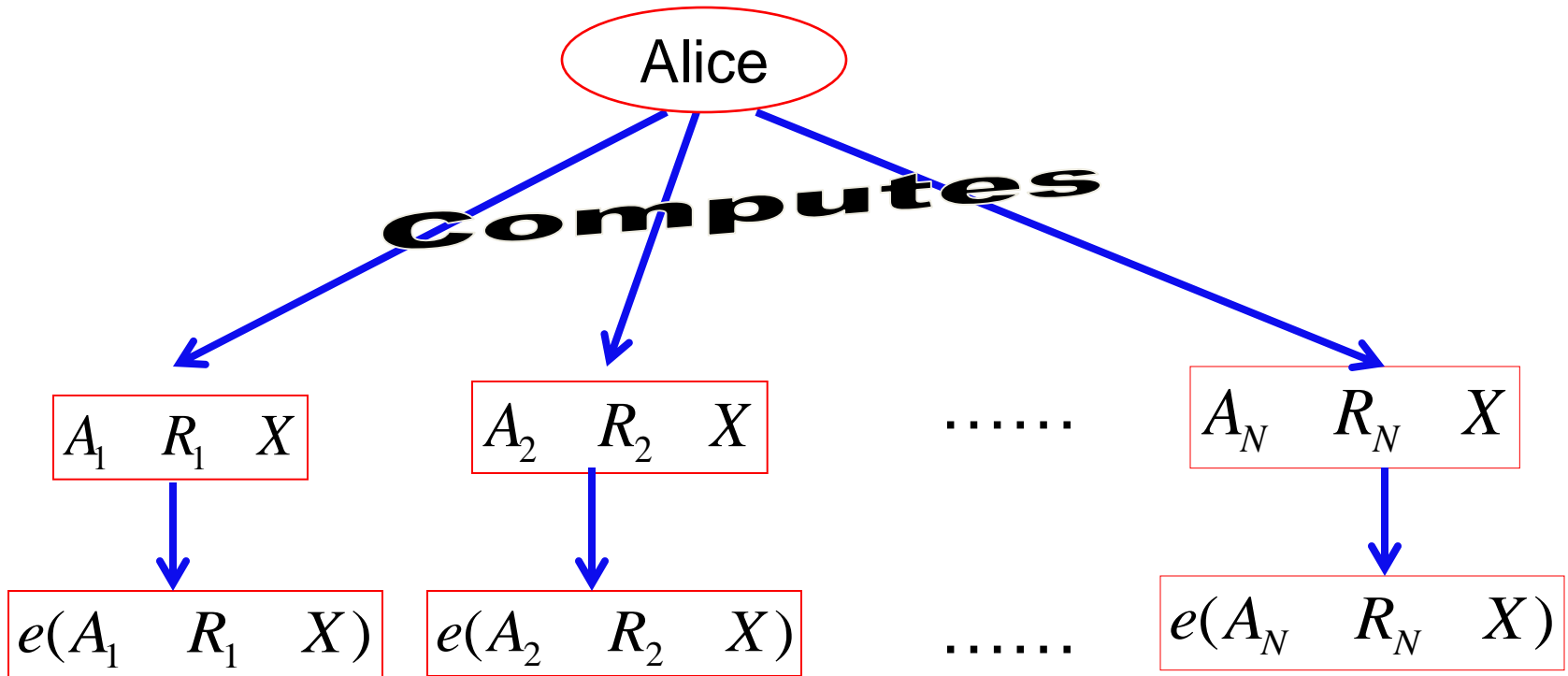


**Digital envelopes**

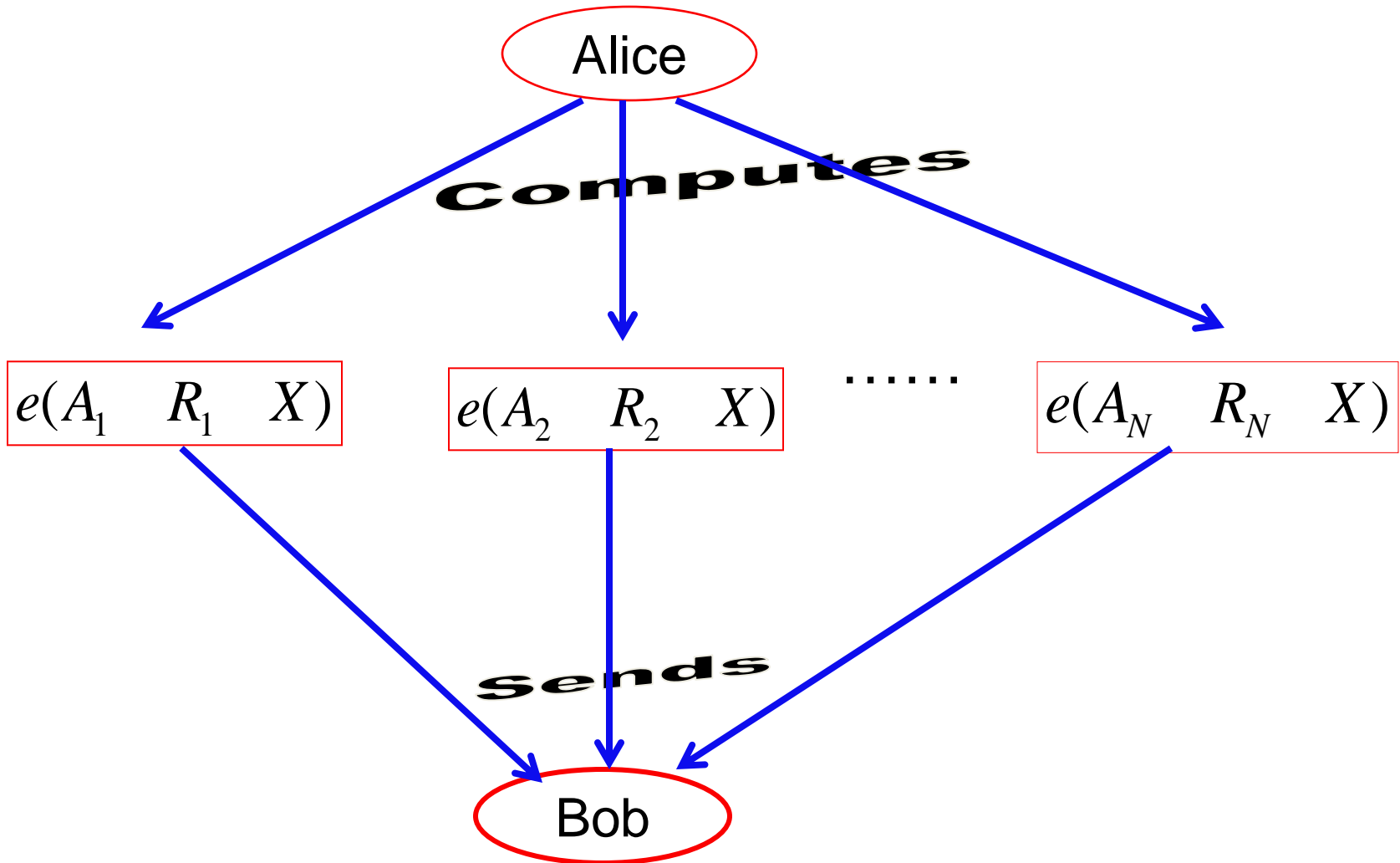
$R_1, R_2, \dots, R_N$

→ A set of random integers generated by Alice

# Step 1



# Step 1



## Step 2

Bob

**computes**

$W_1 \quad e(A_1 \quad R_1 \quad X) \quad B_1$ 
 $W_2 \quad e(A_2 \quad R_2 \quad X) \quad B_2$ 
 $\cdots$ 
 $W_N \quad e(A_N \quad R_N \quad X) \quad B_N$

$B_i \quad 0 \quad W_i \quad 0$

$B_i \quad 1 \quad W_i \quad e(A_i \quad R_i \quad X) \quad B_i \quad e(A_i \quad R_i \quad X)$

## Step 3

- Bob multiplies all the  $w_i$ s for those  $B_i$ s that are not equal to 0. In other words, Bob computes the multiplication of all non-zero  $w_i$ s, e.g.,  $w$  where  $w_i \neq 0$ .

$$w = w_1 \cdot w_2 \cdot \dots \cdot w_j$$

$$W \quad W_1 \quad W_2 \quad \dots \quad W_j$$

$$[e(A_1 \quad R_1 \quad X) \quad \overline{B_1}] \quad [e(A_2 \quad R_2 \quad X) \quad \overline{B_2}] \quad \dots \quad [e(A_j \quad R_j \quad X) \quad \overline{B_j}]$$

$\parallel$   
 $1$

$\parallel$   
 $1$

$\parallel$   
 $1$

$$W \quad W_1 \quad W_2 \quad \dots \quad W_j$$

$$[e(A_1 \quad R_1 \quad X) \quad 1] \quad [e(A_2 \quad R_2 \quad X) \quad 1] \quad \dots \quad [e(A_j \quad R_j \quad X) \quad 1]$$



$$W \quad W_1 \quad W_2 \quad \dots \quad W_j$$

$$e(A_1 \quad R_1 \quad X) \quad e(A_2 \quad R_2 \quad X) \quad \dots \quad e(A_j \quad R_j \quad X)$$



**According to the property of homomorphic encryption**

$$e(A_1 \quad A_2 \quad \dots \quad A_j \quad (R_1 \quad R_2 \quad \dots \quad R_j) \quad X)$$

## Step 4

- Bob generates an integer  $R'$ .
- Bob then computes

$$W' \quad W \quad e(R' \quad X)$$



According to the property of homomorphic encryption

$$e(A_1 \quad A_2 \quad A_j \quad (R_1 \quad R_2 \quad R_j \quad R') \quad X)$$

**sends**

Alice

## The Final Step

- Alice decrypts  $W'$  and computes modulo  $X$ .
- She then obtains the frequency count.

$$c.count \quad d(W') \bmod X$$

# The Final Step

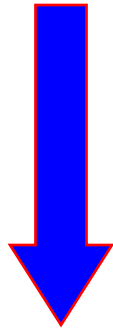
*c.count*

$$d(e(A_1 \quad A_2 \quad A_j \quad (R_1 \quad R_2 \quad R_j \quad R') \quad X)) \bmod X$$

# The Final Step

*c.count*

$(A_1 \ A_2 \ \dots \ A_j \ (R_1 \ R_2 \ \dots \ R_j \ R') \ X) \bmod X$



$(A_1 \ A_2 \ \dots \ A_j) \ N \ X$   
 $((R_1 \ R_2 \ \dots \ R_j \ R') \ X) \bmod X \ 0$

*c.count*

$A_1 \ A_2 \ \dots \ A_j$

# Correctness Analysis

$B_i$	1
$A_i$	1

$c.count$
-----------

$c.count$	$A_1$	$A_2$	$A_j$
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When  $B_i = 1$ ,  $(A_1 \ A_2 \ \dots \ A_j)$  gives the total number of times that both  $A_i$  and  $B_i$  are 1s.

Therefore, the frequency count is correctly computed.

# Privacy Analysis

**Goal: Bob never sees Alice's data values.**

## **Alice's Privacy**

- All the information that Bob obtains from Alice is  $e(A_1 \parallel R_1 \parallel X), e(A_2 \parallel R_2 \parallel X), \dots, e(A_N \parallel R_N \parallel X)$ .
- Since Bob doesn't know the decryption key  $d$ , he cannot get Alice's original data values.

# Privacy Analysis

**Goal: Alice never sees Bob's data values.**

## **Bob's Privacy**

The information that Alice obtains from Bob is

$W' = e(A_1 \parallel A_2 \parallel \dots \parallel A_j \parallel (R_1 \parallel R_2 \parallel \dots \parallel R_j \parallel R') \parallel X)$  for those  $B_i \neq 1$

Alice computes  $d(W') \bmod X$ . She only obtains the frequency count and cannot know Bob's original data values.



# Complexity Analysis

## Communication Cost

Linear in the number of transactions

The total number elements in each attribute vector

$(N - 1)$  where  $N$  is the total number transactions and  $\ell$  is the number of bits for each encrypted element.

# Complexity Analysis

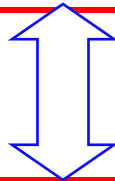
## **Computation Cost**

Linear in the number of transactions

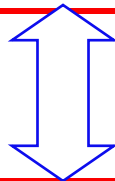
The computational cost is  $(10N + 20 + g)$  where  $N$  is the total number transactions and  $g$  is the computational cost for generating a key pair.

## Other Privacy-Oriented Protocols

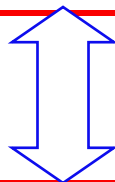
**Multi-Party Frequency Count Protocol**  $\leftarrow - \rightarrow$  [Zhan et al. 2005 (a)]



**Multi-Party Summation Protocol**  $\leftarrow - - - \rightarrow$  [Zhan et al. 2005 (f)]



**Multi-Party Comparison Protocol**  $\leftarrow - - \rightarrow$  [Zhan et al. 2006 (a)]



**Multi-Party Sorting Protocol**  $\leftarrow - - - - \rightarrow$  [Zhan et al. 2006 (a)]

# Our Contributions

- A formal definition of privacy for privacy-preserving collaborative data mining.
- Solutions for data mining tasks for both horizontal collaboration and vertical collaboration.

Association Rule Mining [Zhan et.al.2004(a), Zhan et.al. 2004(b)].

Sequential pattern mining [Zhan et. al.2004(c), Zhan et. al. 2005 (a)].

Naïve Bayesian classification [Zhan et. al.2004(d), Zhan et. al.2005 (b)].

Decision tree classification [Zhan et. al. 2005 (a)-(b)].





k-nearest neighbor classification [Zhan et. al. 2005 (c)-(d)].

Support vector machine classification [Zhan et.al. 2008 (e) - (f)].

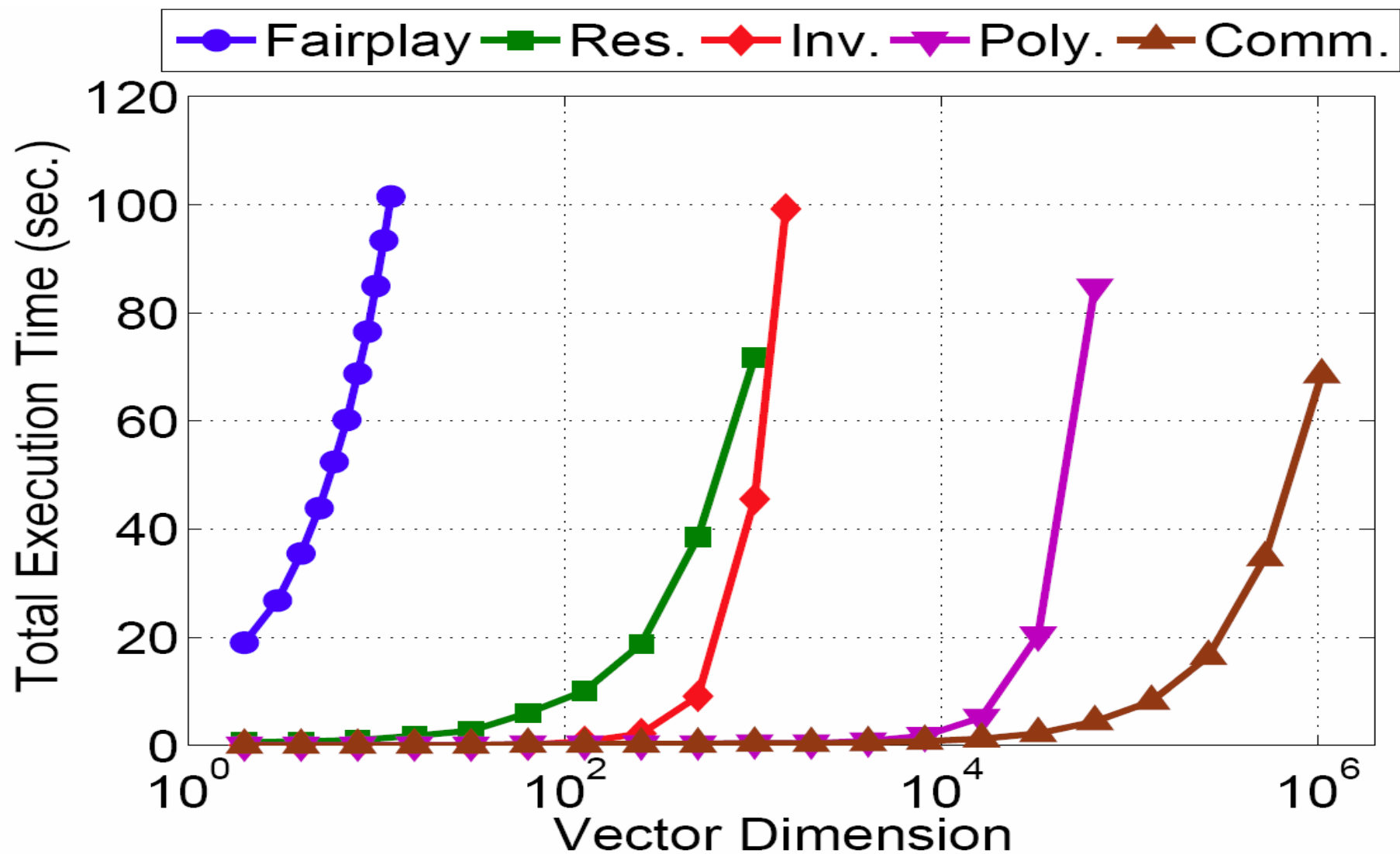
Clustering [Zhan et. al.2005 (g), Zhan et. al. 2008(a)].

- Simulation with various factors including the number of parties involved in the computation, the encryption key size and the size of data set, etc.

# Efficient Privacy-Preserving Collaborative Compiler System Using Scalar Product

<div>Approach</div> <div>Secure</div>	Inv.	Comm.	Poly.	Res.	FP.
Information-theoretically secure					
Computationally secure					

## Execution Time



# Future Works

- Social Computing (IEEE SocialCom)
- <http://www.iisocialcom.org/conference/socialcom2009/>



